Self Evaluation Test – 2

Time Allowed : 1 *Hour* 30 *minutes*] [*Maximum Marks* : 50

1. Find the area bounded by the curve $y = 4\sqrt{x-1}$, x-axis and the line $x = 3$. 1
2. Find the area bounded by the curve $y = 2\cos x$ and the x-axis from $x = 0$ to $x = \pi$. 1
3. Find the area of the parabola $y^2 = 4ax$ bounded by the latus rectum. 6
4. Find the area bounded by the curve $y^2 = 4a^2(x - 3)$ and the lines $x = 3$, $y = 4a$. 6
5. Find the area cut off from the parabola $4y = 3x^2$ by the line $2y = 3x + 12$. 6
6. Find the area of the region :
$$\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}.$$ 6
7. Find the area of the region :
$$\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\}.$$ 6
8. Using integration, find the area of the triangle whose sides have the equations :
$$y = 2x + 1,\ y = 3x + 1,\ x = 4.$$ 6
9. Compute the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$. 6
10. Using integration, find the area of the triangle whose vertices are (2, 0), (4, 5), and (6, 3). 6

ANSWERS

1. $\frac{16\sqrt{2}}{3}$ sq. units
2. 4 sq. units
3. $\frac{8a^2}{3}$ sq. units
4. $\frac{16a}{3}$ sq. units
5. 27 sq. units
6. $\frac{23}{6}$ sq. units
7. $\left(\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}\right)$ sq. units
8. 8 sq. units
9. 6 sq. units
10. 7 sq. units

Self Evaluation Test – 3

Time Allowed : 1 *Hour* 30 *minutes*] [*Maximum Marks* : 50

1. Find the area enclosed between the curve $y = \cos 3x$, $0 \le x \le \frac{\pi}{6}$ and the coordinate axes. 1
2. Write an expression for finding the area of the region bounded by the parabola $y^2 = 16x$ and the line $x = 4$. 1
3. Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 6
4. Find the area included between the curves $x^2 = 4ay$ and $y^2 = 4ax$, $a > 0$. 6
5. Find the area bounded by the curves $y = x$ and $y = x^3$. 6
6. Using integration, find the area of the triangle PQR whose vertices are P (2, 1), Q (3, 4) and R (5, 2). 6
7. Find the area of the region : $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$. 6
8. Find the area of the region : $\{(x, y) : x^2 + y^2 \le 2ax, y^2 \ge ax, x \ge 0, y \ge 0\}$. 6
9. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$, $x = 4$, $y = 0$ and $y = 4$ into three equal parts. 6
10. Draw a rough sketch of the function $y = 2\sqrt{1-x^2}$, $x \in [0, 1]$ and evaluate the area enclosed between the curve and the x-axis. 6

ANSWERS

1. $\frac{1}{3}$ sq. units
2. $2\int_0^4 \sqrt{16x}\,dx$
3. πab sq. units
4. $\frac{16a^2}{3}$ sq. units
5. $\frac{1}{2}$ sq. units
6. 4 sq. units
7. $\left(\frac{\pi - 2}{4}\right)$ sq. units
8. $\left(\frac{\pi}{4} - \frac{2}{3}\right)$ sq. units
10. $\frac{\pi}{2}$ sq. units

Self Evaluation Test – 2

Time Allowed : 1 *Hour*] [*Maximum Marks* : 35

Solve the following differential equations :

1. $\frac{dy}{dx} + \sin(x+y) - \sin(x-y) = 0$. 1

2. $y' = \sec y, y(0) = 0$. 1

3. $\frac{dy}{dx} = -4xy^2$, given $y(0) = 1$. 1

4. Form the differential equation of the family of curves represented by $y^2 - 2ay + x^2 = a^2$, a is arbitrary constant. 4

5. Form the differential equation corresponding to the function $y = ae^{2x} + be^{-2x}$, a and b are arbitrary constants. 4

6. Show that $y = \sin(\sin x)$ is a solution of the differential equation $y'' + (\tan x) \cdot y' + y\cos^2 x = 0$ 4

7. Show that $y^2 = 4a(x+a)$ is a solution of the differential equation $y(1-y_1^2) = 2xy_1$. 4

Solve the following differential equations :

8. $\frac{1}{\sin^{-1} x} \cdot \frac{dy}{dx} = 1$. 4

9. $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$. 6

10. $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}dy = 0$. 6

ANSWERS

1. $\log|\operatorname{cosec} y - \cot y| = -2\sin x + c$ 2. $y = \sin^{-1} x$ 3. $y = \frac{1}{2x^2+1}$

4. $(x^2-2y^2)\left(\frac{dy}{dx}\right)^2 - 4xy\frac{dy}{dx} - x^2 = c$ 5. $y'' - 4y = 0$ 8. $y = x\sin^{-1}x + \sqrt{1-x^2} + c$

9. $\sin\left(\frac{y}{x}\right) = cx$ 10. $xe^x - e^x - \sqrt{1-y^2} = c$

Self Evaluation Test – 3

Time Allowed : 1 *Hour*] [*Maximum Marks* : 35

Find the order and degree of each of the following differential equations :

1. $\left(\frac{dy}{dx}\right)^2 = 7x + \frac{d^2y}{dx^2}$ 1

2. $\cos x\,dx = \sin y\,dy$. 1

3. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$. 1

4. Form the differential equation corresponding to the function $y = 2(x^2-1) + ce^{-x^2}$. 4

5. Verify that $y + x + 1 = 0$ is a solution of the differential equation $(y-x)\,dy - (y^2-x^2)\,dx = 0$. 4

Solve the following differential equations :

6. $(e^x + e^{-x})\,dy = (e^x - e^{-x})\,dx$. 4

7. $xy' + y = x\cos x + \sin x, y\left(\frac{\pi}{2}\right) = 1$. 4

8. $y\,dx + (x-y^3)dy = 0$. 4

9. $(\sin y + y\cos y)\frac{dy}{dx} = 2x(\log x + 1)$, when $y(1) = 0$. 6

10. Show that the family of curves for which the slope of the tangent at any point (x, y) on it is $\frac{x^2+y^2}{2xy}$, is given by $x^2 - y^2 = cx$. 6

ANSWERS

1. Order = 2, degree = 1 2. Order = 1, degree = 1 3. Order = 2, degree = 2

4. $\frac{dy}{dx} + 2xy = 4x^3$ 6. $y = \log|e^x + e^{-x}| + c$ 7. $y = \sin x$

8. $xy = \frac{1}{4}y^4 + c$ 9. $y\sin y = x^2\log x + c$