

TOPIC: BINOMIAL THEOREM.

- Using binomial theorem, evaluate  $(99)^4$ .
- Write the binomial expansion of  $(1+x)^{n+1}$ ,  $n \in \mathbb{N}$  and hence deduce that  $9^{n+1} - 8n - 9$  is divisible by 64 for all  $n \in \mathbb{N}$ .
- Find the middle term in the expansion of  $\left(\frac{2x^2}{3} - \frac{3}{2x}\right)^{12}$ .
- Find the coefficient of  $x^{-17}$  in the expansion of  $\left(x^4 - \frac{1}{x^3}\right)^{15}$ .
- If the coefficients of 5th, 6th, 7th terms in the expansion of  $(1+x)^n$  are in A.P., find  $n$ .
- Expand:  $(1-x+x^2)^4$ .
- Find  $a$ , if the 17th and 18th terms in the expansion of  $(2+a)^{50}$  are equal.
- The binomial coefficient of the third term from the end in the expansion of  $(y^{2/5} + x^{5/4})^n$  is 91. Find the 9th term of the expansion.
- Prove that in the middle term in the expansion of  $(1+x)^{2n}$  is  $\frac{1.3.5.....(2n-1)}{n!} 2^n \cdot x^n$ .
- The sum of the coefficients of first three terms in the expansion of  $\left(x - \frac{3}{x^2}\right)^n$ ,  $x \neq 0$ ,  $n \in \mathbb{N}$  is 559. Find the coefficient of  $x^3$  in the expansion.

- The first three terms in the expansion of  $(1+ax)^n$  are 1,  $12x$  and  $64x^2$  respectively. Find  $n$  and  $a$ . 3
- Evaluate:  $(2 + \sqrt{3})^7 - (2 - \sqrt{3})^7$ , using binomial theorem. 4
- Find the middle term in the expansion of  $\left(x - \frac{1}{2y}\right)^{10}$ . 3
- Find the term independent of  $x$  in the expansion of  $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ . 4
- If  $a_1, a_2, a_3, a_4$  be the coefficients of four consecutive terms in the expansion of  $(1+x)^n$ , then prove that: 3
 
$$\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}$$
- Find the greatest term in the expansion of  $(1+2x)^8$  when  $x = 2$ . 3
- Which number is larger  $(1.2)^{4000}$  or 800? 4
- If the coefficients of  $x^{r-1}, x^r, x^{r+1}$  in the expansion of  $(1+x)^n$  are in A.P. Prove that, 4
 
$$n^2 - (4r+1)n + 4r^2 - 2 = 0$$
- Prove that the coefficient of one middle term in the expansion of  $(1+x)^{2n}$  is equal to the sum of the coefficients of two middle terms in the expansion of  $(1+x)^{2n-1}$ . 3

TOPIC : SEQUENCE AND SERIES.

- Find the 15th term from the end of the sequence 7, 10, 13, ... 130.
- If  $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$  are in A.P., show that  $\frac{a}{c} = \frac{a-b}{b-c}$ .
- How many terms of the A.P. 17, 15, 13 ... must be taken so that the sum is 72 ? Explain the double answer.
- In an increasing G.P., the sum of first and the last term is 66, the product of the second and the last but one term is 128. If the sum of the series is 126, find the number of terms of the series.
- Let the sequence  $\{a_n\}$  be defined as follows :  
 $a_1 = 1; a_n = a_{n-1} + 2$  for  $n \geq 2$   
 Find the first five terms of the series.
- The income of Mehul is Rs. 3,00,000 in the first year and he receives an increase of Rs. 1,000 per year. Find the total amount he received in 20 years.
- If G is geometric mean between  $a$  and  $b$ , show that  $\frac{1}{G+a} + \frac{1}{G+b} = \frac{1}{G}$ .
- The difference between any two consecutive interior angles of a polygon is  $5^\circ$ . If the smallest angle is  $120^\circ$ , find the number of sides of the polygon.
- If  $a, b, c$  are in G.P. and  $a^x = b^y = c^z$ , prove that  $x, y, z$  are in A.P.
- Find the sum of first  $n$ -terms of the series :  
 $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots$

- For what value of  $n$ , the  $n$ th terms of the series :  
 "3 + 10 + 17 + ..." and "63 + 65 + 67 + ..." are equal.
- If  $a, b$  are the roots of  $x^2 - 3x + p = 0$  and  $c, d$  are the roots of  $x^2 - 2x + q = 0$ , where  $a, b, c, d$  form a G.P. Prove that  $(q+p) : (q-p) = 17 : 15$ .
- If  $S_n$  denotes the sum of  $n$  terms of an A.P. and if  $S_1 = 6, S_7 = 105$ , show that  $\frac{S_n}{S_{n-3}} = \frac{n+3}{n-3}$ .
- If  $a, b, c, d$  are in G.P., show that  $a^2 + b^2 + c^2, ab + bc + cd, b^2 + c^2 + d^2$  are also in G.P.
- If  $S_n$  denotes the sum of  $n$  terms of a G.P., prove that  $(S_{10} - S_{20})^2 = S_{10} (S_{30} - S_{20})$ .
- Find the sum  $4 + 0.4 + 0.44 + 0.444 + \dots$   $n$  terms.
- If A and G be respectively A.M. and G.M. between two positive numbers, prove that the numbers are  $A \pm \sqrt{A^2 - G^2}$ .
- If  $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$  ( $x \neq 0$ ) then show that  $a, b, c, d$  are in G.P.
- Find the sum of the series :  $18^2 + 19^2 + \dots + 47^2$ .